

Mathematical Logics

Modal Logic: Introduction*

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1. Intuition
2. Language
3. Relational structures and Satisfiability
4. Validity, unsatisfiability, Logical consequence and equivalence

The Language of a propositional modal logic

If P is a set of primitive proposition, the set of formulas of the basic modal logic is defined as follows:

- each $p \in P$ is a formula (atomic formula);
- if A and B are formulas then $\neg A$, $A \wedge B$, $A \vee B$, $A \supset B$ and $A \equiv B$ are formulas
- if A is a formula $\Box A$ and $\Diamond A$ are formulas.

NOTE: there are also first order modal logics (not covered in this class)

Intuitive interpretation of modal logic

The formula $\Box\varphi$ can be intuitively interpreted in many ways

- φ is necessarily true (classical modal logic)
- φ is known/believed to be true (epistemic logic)
- φ is provable in a theory (provability logic)
- φ will be always true (temporal logic)
- ...

In all these cases $\Diamond\varphi$ is interpreted as $\neg\Box\neg\varphi$.

In other words, $\Diamond\varphi$, stands for $\neg\varphi$ is not necessarily true, that is, φ is possibly true.

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