Mathematical Logics

FOL: Reasoning as deduction

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Lecture index

1. Reasoning problems (recap)
2. Hilbert systems (VAL – forward chaining)
3. Tableaux systems ((un)-SAT – backward chaining)
4. Correctness and completeness of Tableau
5. Examples
6. Termination
7. Countermodels

Some definition for tableaux

Definition (Closed branch)

A closed branch is a branch which contains a formula and its negation.

Definition (Open branch)

An open branch is a branch which is not closed

Definition (Closed tableaux)

A tableaux is closed if all its branches are closed.

Definition (logical consequence via Tableaux)

Let φ be a first-order formula and Γ a finite set of such formulas. We write $\Gamma \vdash \varphi$ to say that there exists a closed tableau for $\Gamma \cup \{\neg\varphi\}$

Correctness and completeness

Theorem (Correctness / soundness)

$$\Gamma \vdash \varphi \quad \Rightarrow \quad \Gamma \models \varphi$$

Theorem (completeness)

$$\Gamma \models \varphi \quad \Rightarrow \quad \Gamma \vdash \varphi$$

Remark 1

Proof: trivial, follows directly from definition. Consequence of refutation theorem (holding also for FOL)

Remark 2

The mere existence of a closed tableau does not mean that we have an effective method to build it! Concretely: we don't know how often and in which way we have to apply] the γ-rules ($\forall x \varphi(x) \Rightarrow \varphi[x/t]$), and what term to use in the substitution.

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