Mathematical Logics

FOL: Reasoning as deduction

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Lecture index

1. Reasoning problems (recap)
2. Hilbert systems (VAL – forward chaining)
3. Tableaux systems ((un)-SAT – backward chaining)
4. Correctness and completeness of Tableau
5. Examples
6. Termination
7. Countermodels

Practicing with Tableaux

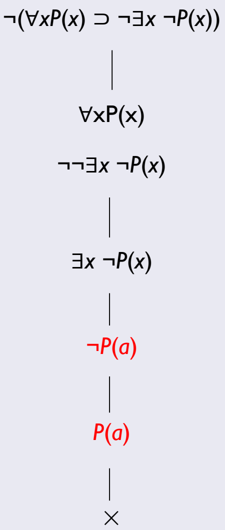
Exercize

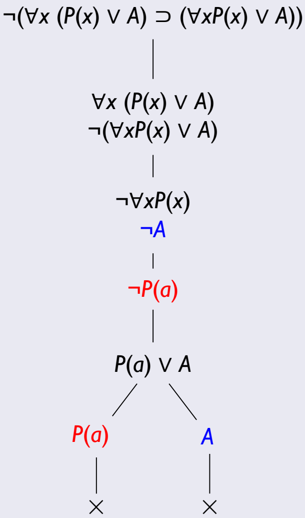
Show with the method of semantic tableaux that the following formulas are valid:

- $\forall x P(x) \supset \neg \exists x \neg P(x)$
- $\forall x\, (P(x) \vee A) \supset (\forall x P(x) \vee A)$ when x is not free in A
- $\exists x\, (P(x) \supset \forall x P(x))$
- $\exists x\, \forall y P(x, y) \supset \forall y\, \exists x P(x, y)$

Practicing with Tableaux

Solution





Practicing with Tableaux

Solution

$$\neg\exists x\, (P(x) \supset \forall x P(x))$$
$$|$$
$$\neg(P(a) \supset \forall x P(x))$$
$$|$$
$$P(a)$$
$$\neg\forall x P(x)$$
$$|$$
$$\neg P(b)$$
$$|$$
$$\neg(P(b) \supset \forall x P(x))$$
$$|$$
$$P(b)$$
$$\neg\forall x P(x)$$
$$|$$
$$\times$$

$$\neg(\exists x\, \forall y P(x, y) \supset \forall y\, \exists x P(x, y))$$
$$|$$
$$\exists x\, \forall y P(x, y)$$
$$\neg\forall y\, \exists x P(x, y)$$
$$|$$
$$\forall y P(a, y)$$
$$|$$
$$\neg\exists x P(x, b)$$
$$|$$
$$P(a, b)$$
$$|$$
$$\neg P(a, b)$$
$$|$$
$$\times$$

Tableaux

Exercize

Give tableau proofs for the following logical consequences:

- $\forall x.P(x) \lor \forall x.Q(x) \models \neg\exists x\,(\neg P(x) \land \neg Q(x))$
- $\models \exists x.(P(x) \lor Q(x)) \equiv \exists x.P(x) \lor \exists x.Q(x)$

Prove the following FOL properties of quantifiers

Proposition

The following formulas are valid

- $\forall x\,(\varphi(x) \land \psi(x)) \equiv \forall x\varphi(x) \land \forall x\psi(x)$
- $\exists x\,(\varphi(x) \lor \psi(x)) \equiv \exists x\varphi(x) \lor \exists x\psi(x)$
- $\forall x\varphi(x) \equiv \neg\exists x\,\neg\varphi(x)$
- $\forall x\,\exists x\varphi(x) \equiv \exists x\varphi(x)$
- $\exists x\,\forall x\varphi(x) \equiv \forall x\varphi(x)$

Proposition

The following formulas are not valid (prove the correct direction)

- $\forall x\,(\varphi(x) \lor \psi(x)) \equiv \forall x\varphi(x) \lor \forall x\psi(x)$
- $\exists x\,(\varphi(x) \land \psi(x)) \equiv \exists x\varphi(x) \land \exists x\psi(x)$
- $\forall x\varphi(x) \equiv \exists x\varphi(x)$
- $\forall x\,\exists y\varphi(x, y) \equiv \exists y\,\forall x\varphi(x, y)$

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