

Mathematical Logics

Modal Logic: K and more*

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1. **Calculi for modal logics**
2. Modal K (Hilbert calculus)
3. Properties of accessibility relation and modal axioms
4. Modal KT
5. Modal KB
6. Modal KD
7. Modal $KT4 = S4$
8. Modal $KT5 = S5$
9. MultiModal Logics
10. Multiagent Knowledge and belief

- **Hilbert calculi:** used for properties of logics
- **Tableau calculus:** used for reasoning
 - *This Lecture:* the family of propositional modal logics
 - *Next Lecture:* Tableaux calculus

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7. Modal $KT4 = S4$
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9. Recap
10. MultiModal Logics
11. Multiagent Knowledge and belief

Hilbert axioms for the normal modal logic K

A1	$\varphi \supset (\psi \supset \varphi)$
A2	$(\varphi \supset (\psi \supset \vartheta)) \supset ((\varphi \supset \psi) \supset (\varphi \supset \vartheta))$
A3	$(\neg\psi \supset \neg\varphi) \supset ((\neg\psi \supset \varphi) \supset \varphi)$
MP	$\frac{\varphi \quad \varphi \supset \psi}{\psi}$
K	$\Box(\varphi \supset \psi) \supset (\Box\varphi \supset \Box\psi)$
Nec	$\frac{\varphi}{\Box\varphi}$ the necessitation rule

Necessitation rule can be applied only if premise is a theorem

The above set of axioms and rules is called **K**, and every modal logic with a validity relation closed under the rules of **K** is a **Normal Modal Logic**.

Notice that **Nec** rule is not the same as

$$\varphi \supset \Box \varphi \tag{3}$$

indeed formula (3) is not valid.

Assignment Find a model in which (3) is false. Hint: build model with a model where φ is true and $\Box \varphi$ is false (namely $\neg \Box \varphi$ is true)

Nec: In Hilbert calculi there is difference from deduction and proof. Deduction theorem does not hold

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