

Mathematical Logics

Description Logic: Tableaux

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1. Idea: DL is a MultiModal Modal Logic
2. DL reasoning as MultiModal SAT reasoning
3. Examples: TBOX reasoning
4. Examples: ABOX reasoning – DL as a query language

DL & MultiModal Modal Logics - intuition

- ❑ An entity is a world
- ❑ An entity is capable of propositional reasoning
- ❑ A Relation is an accessibility relation (e.g., *friend(Fausto, Mattia)* means that Mattia is accessible to Fausto via friendship; *talksTo(Fausto, Mattia)* means that Mattia is accessible to Fausto via talking)
- ❑ A data type is a set of entities whose behavior is known a priori and computed by the reasoner (via the data type operations, equality, recognizer) (e.g., *Height(Fausto, 175)* same as *Height(Fausto, 170+5)*)
- ❑ An attribute is an accessibility relation to a data type
- ❑ All the relations/ attributes which have an entity in their domain define all possible ways in which that entity has access to the other world entities
- ❑ An etype is a set of worlds (entities) which have the same set of accessibility relations
- ❑ $\exists R.C$ means accessibility to at least another entity or etype C
- ❑ $\forall R.C$ means accessibility to all entities of etype C (in other words: to **ONLY** entities of etype C)

DL & MultiModal Modal Logics – Language mapping

- \neg same as \neg
- \sqcap / \sqcup same as \wedge / \vee
- \sqsubseteq same as \rightarrow
- entities are worlds: a, b, c, \dots
- TBOX: worlds made explicit in the language as variables
 - Etype C written as $C(x)$
 - Relation R written as $R(x, y)$
 - $\exists R.C$ same as $\diamond_R C$ written as $\exists R.C(x)$
 - $\forall R.C$ same as $\square_R C$ written as $\forall R.C(x)$
- ABOX: worlds made explicit in the language as constants
 - Etype C written as $C(a)$
 - Relation R written as $R(a, b)$
 - $\exists R.C$ same as $\diamond_R C$ written as $\exists R.C(a)$
 - $\forall R.C$ same as $\square_R C$ written as $\forall R.C(a)$

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