Mathematical Logics
Description Logic: Tableaux

Fausto Giunchiglia and Mattia Fumagallli

University of Trento

**Originally by Luciano Serafini and Chiara Ghidini*
Modified by Fausto Giunchiglia and Mattia Fumagalli

Lecture index

1. Idea: DL is a MultiModal Modal Logic
2. DL reasoning as MultiModal SAT reasoning
3. Examples: TBOX reasoning
4. Examples: ABOX reasoning – DL as a query language

Example of Tableau Reasoning (1)

- Is $\forall hasChild.Male \sqcap \exists hasChild.\neg Male$ satisfiable?

  NOTE: we do not have an initial T or A

  $\sqcap$-rule

  A = {($\forall$hasChild.Male)(x), ($\exists$hasChild.¬Male)(x)}

  $\exists$-rule

  A = {($\forall$hasChild.Male)(x), hasChild(x,y), ¬Male(y)}

  $\forall$-rule

  A = {($\forall$hasChild.Male)(x), hasChild(x,y), ¬Male(y), Male(y)}

A is clearly inconsistent and therefore the formula is not satisfiable.

Example of Tableau Reasoning (2)

- Suppose T = {A $\sqsubseteq$ B} and A = {}

Is $\exists R.(A \sqcap C) \sqcap \forall R.(\neg B \sqcap C)$ satisfiable?

$\sqcap$-rule A = {$\exists R.(A \sqcap C)(x)$, $\forall R.(\neg B \sqcap C)(x)$}

$\exists$-rule A = {$R(x, y)$, $(A \sqcap C)(y)$, $\forall R.(\neg B \sqcap C)(x)$}

$\sqcap$-rule A = {$R(x, y)$, $A(y)$, $C(y)$, $\forall R.(\neg B \sqcap C)(x)$}

$\forall$-rule A = {$R(x, y)$, $A(y)$, $C(y)$, $(\neg B \sqcap C)(y)$}

$\sqcap$-rule A = {$R(x, y)$, $A(y)$, $C(y)$, $\neg B(y)$}

By expanding A w.r.t. T we discover that it is inconsistent. Therefore the formula is not satisfiable.

Example of Tableau Reasoning (3.a)

- Is $\exists S.C \sqcap \forall S.(\neg C \sqcup \neg D) \sqcap \exists R.C \sqcap \forall R.(\exists R.C)$ satisfiable?

NOTE: we do not have an initial T or A

$\sqcap$-rule A = {$\exists S.C(x)$, $\forall S.(\neg C \sqcup \neg D)(x)$, $\exists R.C(x)$, $\forall R.(\exists R.C)(x)$}

$\exists$-rule A = {$S(x, y)$, $C(y)$, $\forall S.(\neg C \sqcup \neg D)(x)$, $\exists R.C(x)$, $\forall R.(\exists R.C)(x)$}

$\forall$-rule A = {$S(x, y)$, $C(y)$, $(\neg C \sqcup \neg D)(y)$, $\exists R.C(x)$, $\forall R.(\exists R.C)(x)$}

$\sqcup$-rule

A' = {$S(x, y)$, $C(y)$, $\neg C(y)$, $\exists R.C(x)$, $\forall R.(\exists R.C)(x)$} (clash!) or

A'' = {$S(x, y)$, $C(y)$, $\neg D(y)$, $\exists R.C(x)$, $\forall R.(\exists R.C)(x)$}

Example of Tableau Reasoning (3.b)

A” = {S(x, y), C(y), ¬D(y), ∃R.C(x), ∀R.(∃R.C)(x)}

∃-rule A” = {S(x, y), C(y), ¬D(y), R(x, z), C(z), ∀R.(∃R.C)(x)}

∀-rule A” = {S(x, y), C(y), ¬D(y), R(x, z), C(z), ∃R.C(z)}

∃-rule A” = {S(x, y), C(y), ¬D(y), R(x, z), C(z), R(z, w), C(w)}

A” is satisfiable

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