

Computational Logic Exercises - SetTheory

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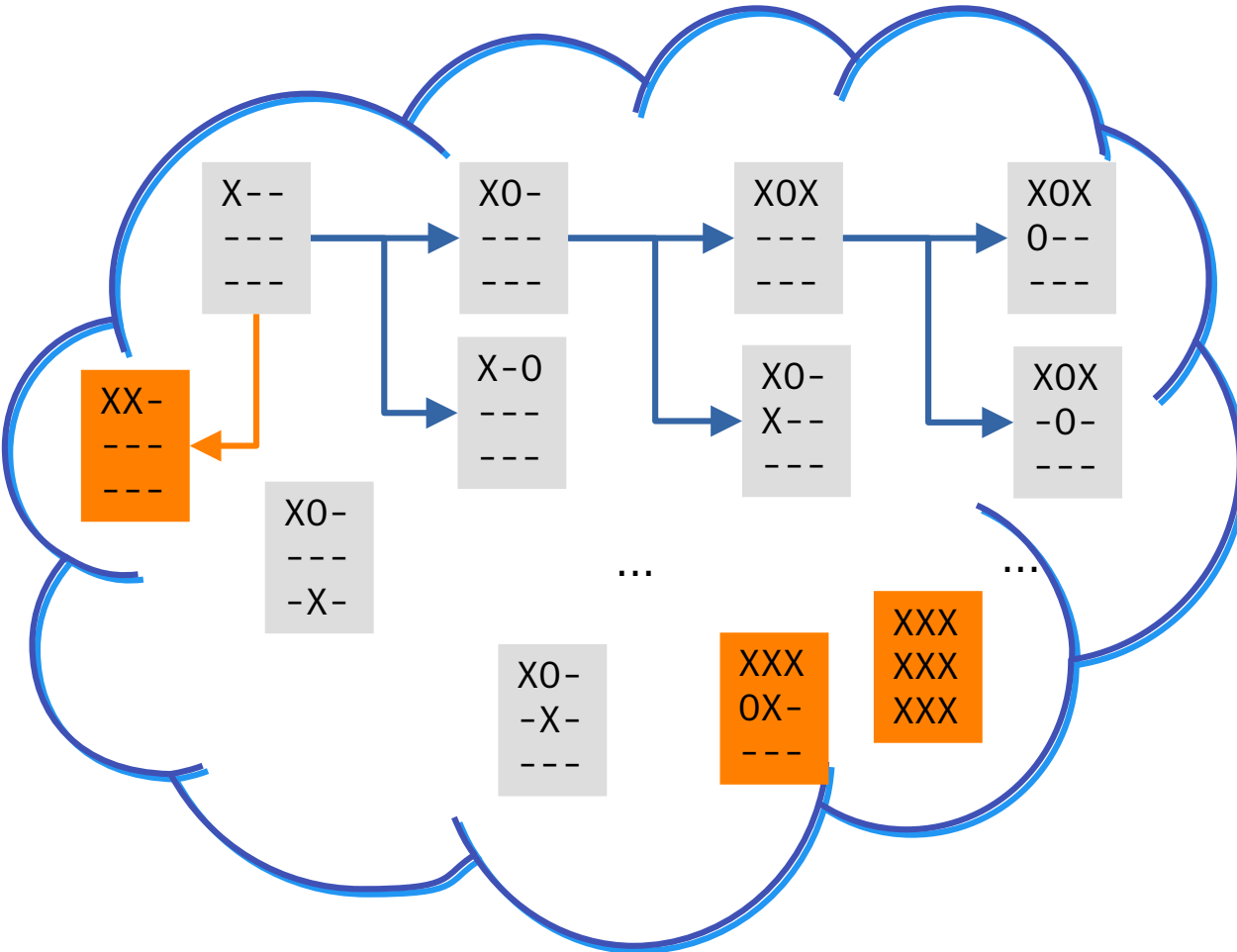


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Exercise at Home

- Represent the “next move” relationship in Tic-Tac-Toe
- Start from the set of board positions
- What properties does it have? Why (or why not)?
 - Reflexive?
 - Symmetric?
 - Transitive?
 - Anti-symmetric?
 - Surjective?
 - Injective?
 - Can you find a partial order?
 - Can you find a partition?

nextMove



- Domain: sets of all board states (both “possible” and “impossible” states)
- Next move: binary relationship between two states (current board, next board)
- It is up to us to decide whether to define it only on “good” moves or not (however, we would then have the “issue” of proving its various properties, unless we partition the set)

NextMove: Reflexive?

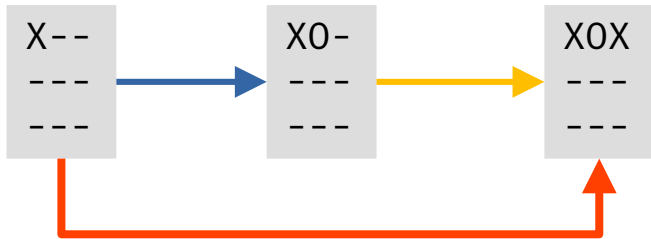
- reflexive iff $\text{nextMove}(a, a)$ for all $a \in D$
- NextMove is not reflexive
- It would be only if we had a “pass” in the game

nextMove: Symmetric?

- symmetric iff
nextMove(a, b)
implies nextMove(b,
a) for all a, b $\in D$
- No
- A relationship
“connected”
representing both
next and previous
move could be.

nextMove: Transitive?

- transitive iff
 $\text{nextMove}(a, b)$ and
 $\text{nextMove}(b, c)$ imply
 $\text{nextMove}(a, c)$ for all
 $a, b, c \in D$

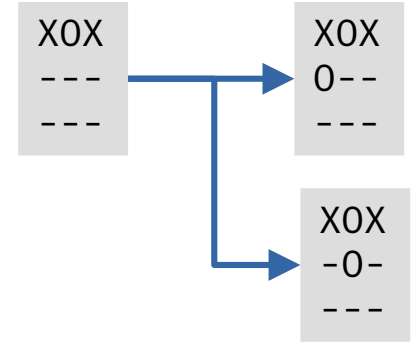


- No
- A “reachable” relationship is (reachable = can I reach this state from this other one?)

nextMove: Anti-symmetric?

- anti-symmetric iff $\text{nextMove}(a, b)$ and $\text{nextMove}(b, a)$ imply $a = b$ for all $a, b \in D$
- No, since we don't have a and b such that $(\text{nextMove}(a, b)$ and $\text{nextMove}(b, a))$ is true

nextMove: function?



- Surjective and injective are properties of functions. We need to ask first whether nextMove is a function, then.
- A function f from A to B is a relation that associates to each element a in A exactly one element b in B . Denoted with $f : A \rightarrow B$
- No.
- Intuitively: given a board state there are different possible moves and, therefore, different possible next states

Examples of Functions in D?

- $F : D \rightarrow \{0..9\}$
(numbers of square filled)
- Surjective, not injective
- $G : D \rightarrow \{X_Wins, Draw, Y_Wins, Intermediate, Impossible\}$
- Surjective, not injective

Defining an Order

- We need to find a relation which is:
 - Reflexive
 - Anti-symmetric
 - Transitive
- $a R b$ if the number of filled cells in a is less or equal to the number of filled cells in b
- (R measures how far we are in the game)

Partitions

- Let A be a set, a partition of A is a family F of non-empty subsets of A s.t.: the subsets are pairwise disjoint the union of all the subsets is the set A
 - Many, many, many:
 - The subset of the legal boards and the set of the illegal boards are a partition of D
 - In addition to previous: the legal boards are further divided in the set of final states and that of non-final states
 - In addition to previous: the set of final states in which X -wins, those in which O -wins, those in which none wins and all non-final states
 - The set of all boards with the first row empty and the set of boards with the first row non empty
 - ...

Good luck!

Questions:

<https://github.com/avillafiorita/cl-2020>

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